We organize a workshop in person and through zoom. Registration is mandatory through the link below.

Mini-workshop : Global Analysis and Geometry

(1) Date (in Japan Standard Time (JST)) :

March 29, 2023 (Wednesday) 13:00 $\longrightarrow$ 17:15 + (a social gathering)

March 30, 2023 (Thursday) 10:00 $\longrightarrow$ 12:00 $\overset{\text{lunch break}}{\longrightarrow}$ 13:30 $\longrightarrow$ 17:35

(2) Place : Osaka Central Advanced Mathematical Institute (OCAMI), Osaka Metropolitan University
Auditorium Place : Room F415, Building F

(3) Participation : On-site and on-line
In both cases, please make registration through

`https://forms.gle/M8i32E6cGEdysUcy9`

no later than March 27th, 2023. The Zoom link will be send March 28th.

(4) Organizers : Tarama Daisuke (Ritsumeikan Univ.), Furutani Kenro (OCAMI), Homma Yasushi (Waseda Univ.)

(5) Supports : JSPS KAKENHI Grant Number 19K03480, 19K14540, 20K03662 and Osaka Central Advanced Mathematical Institute, Osaka Metropolitan University

OCAMI

科研費 KAKENHI

Invited Speakers

Name	University	Country	on-line/on-site
Adachi Jiro	Hokkaido University	Japan	on-site
Wolfram Bauer	Leibniz University Hannover	Germany	on-line
Erlend Grong	University of Bergen	Norway	on-line
Iwasaki Chisato	University of Hyogo	Japan	on-site
Homma Yasushi	Waseda University	Japan	on-site
Irina Markina	University of Bergen	Norway	on-site
Morimoto Tohru	Oka Mathematical Institute, Nara Women's Univ.	Japan	on-site
Ohnita Yoshihiro	OCAMI, Osaka Metropolitan University	Japan	on-site
Ohno Soma	Waseda University	Japan	on-site
Tamaru Hiroshi	OCAMI, Osaka Metropolitan University	Japan	on-site

Program (in JST)

2023/03/29 (Wednesday)	**Speaker** & **Title**
13:00 -13:10	**Opening**
13:10 -14:00	**Iwasaki Chisato** (Univ. Hyogo) Symbolic calculus of pseudo-differential operators on manifolds with conical singularities
14:10 -15:00	**Ohnita Yoshihiro** (OCAMI, Osaka Metropolitan Univ.) Geometry of Lagrangian submanifolds and isoparametric submanifolds
15:00 -15:25	Tea time
15:25 -16:15 8:25 -9:15 (CET)	**Wolfram Bauer** (Leibniz Univ. Hannover) Subriemannian structures on $\mathbb{S}^7$ and beyond
16:25 -17:15 9:25 -10:15 (CET)	**Erlend Grong** (Univ. Bergen) Sub-Riemannian geometry, most probable paths and transformations
18:30 –	(social gathering)

2023/03/30 (Thursday)	**Speaker** & **Title**
10:00-10:50	**Irina Markina** (Univ. Bergen/OCAMI) On H-type algebras: history, progress, and perspectives
11:00-11:50	**Tamaru Hiroshi** (OCAMI, Osaka Metropolitan Univ.) On the moduli spaces of left-invariant Riemannian metrics on Lie groups
12:00-13:20	Lunch
13:20-14:10	**Morimoto Tohru** (Oka Mathematical Institute, Nara Women's Univ.) Three dimensional contact submanifolds of special interest in the seven dimensional projective space and the associated systems of linear PDE's of second order
14:20-15:10	**Adachi Jiro** (Hokkaido Univ.) Existence and Classification of certain non-integrable distributions
15:10-15:30	Tea time
15:30-16:20	**Ohno Soma** (Waseda Univ.) Rarita-Schwinger fields on nearly parallel G_2-manifolds
16:30-17:20	**Homma Yasushi** (Waseda Univ.) Higgs algebra in harmonic analysis on the Grassmannian of 2-planes
17:25 -17:35	**Closing**

Titles and Abstracts

2023/03/23 (Thursday)

(1) 13:10–14:00 **Chisato Iwasaki** (岩崎千里), University of Hyogo

Title : **Symbolic calculus of pseudo-differential operators on manifolds with conical singularities**

Abstract : Symbolic calculus of pseudo-differential operators on $\mathbf{R^n}$ is an essential tool for construction of the fundamental solution to the heat equation on smooth manifolds. In this talk I will show the modified Bessel function plays an important role in asymptotic form of the fundamental solution to the heat equation on manifolds with conical singularities.

This talk is based on the joint study with Kenro Furutani and Wolfram Bauer.

(2) 14:10–15:00 **Ohnita Yoshihiro** (大仁田義裕), OCAMI, Osaka Metropolitan University

Title : Geometry of Lagrangian submanifolds and isoparametric submanifolds

Abstract :

15:00 – 15:25 **Tea time**

(3) 15:25–16:15 **Wolfram Bauer**, Leibniz University Hannover

Title : Subriemannian structures on $\mathbb{S}^7$ and beyond

Abstract : In this talk we compare various subriemannian structures defined on the seven dimensional Euclidean sphere. These are interesting examples in subriemannian geometry, which - under geometric and analytic aspects - can be analyzed rather explicitly. In particular, we study their subriemannian isometry groups and address the question of complete integrability of the subriemannian geodesic flow. From an analytic perspective we discuss the spectra of the induced hypoelliptic sublaplacians as well as asymptotic properties of the heat kernel. If time permits related results on the heat kernel asymptotic for H-type foliations will be mentioned. This is a class of manifolds generalizing some of the above cases. Our presentation surveys results obtained in former and ongoing joint work with K. Furutani, C. Iwasaki, A. Laaroussi, I. Markina, D. Tarama and G. Vega-Molino.

(4) 16:25-17:15 **Erlend Grong**, University of Bergen

Title : **Sub-Riemannian geometry, most probable paths and transformations.**

Abstract : Doing statistics on a Riemannian manifold becomes very complicated for the reason that we lack pluss to define such things as mean and variance. Using the Riemannian distance, we can define a mean know as the Fréchet mean, but this gives no concept of asymmetry, also known as anisotropy. We introduce an alternative definition of mean called the diffusion mean, which is able to both give a mean and the analogue of a covariance matrix for a dataset on a Riemannian manifolds. Surprisingly, computing this mean and covariance is related to sub-Riemannian geometry. We describe how sub-Riemannian geometry can be applied in this setting, and mention some finite dimensional and infinite-dimensional applications.

The results are part of joint work with Stefan Sommer (Copenhagen, Denmark)

2023/03/30 (Thursday)

(5) 10:00–10:50 **Irina Markina**, University of Bergen, Norway/OCAMI

Title : **On H-type algebras: history, progress, and perspectives**

Joint work with K.Furutani, Osaka Central Advanced Mathematical Institute, Osaka Metropolitan University, Osaka, Japan.

Abstract : In this talk, I will concentrate on the joint long-lasting project with Professor Furutani. I start from historical notes: who and why introduced H-type Lie algebras, which are the natural generalization of the Heisenberg Lie algebra. We will reveal the relation of these Lie algebras with Clifford algebras. After mentioning some known properties, we will concentrate on the discussion of the existence of lattices on the corresponding H-type Lie groups, and their possible classification which is invariant under group automorphisms.

(6) 11:00–11:50 **Tamaru Hiroshi** (田丸博士), OCAMI, Osaka Metropolitan University

Title : **On the moduli spaces of left-invariant Riemannian metrics on Lie groups**

Abstract : For a given Lie group, a certain quotient space of the space of all left-invariant Riemannian metrics is called the moduli space. Properties of the moduli space, such as the dimension or topological type, are invariants of the Lie group, and could be expected to reflect properties of left-invariant metrics. In this talk, we introduce this framework, and mention some recent studies on Lie groups whose moduli spaces are one-dimensional.

12:00–13:20 **Lunch**

(7) 13:20–14:10 **Morimoto Tohru** (森本徹), Oka Mathematical Institute, Nara Women's University

Title : **Three dimensional contact submanifolds of special interest in the seven dimensional projective space and the associated systems of linear PDE's of second order**

Abstract : We like to present some special concrete examples of contact submanifolds in projective space as a consequence of our general classification program and discuss their symmetries and properties in relation to the associated differential equations

(8) 14:20–15:10 **Adachi Jiro** (足立二郎), Hokkaido University

Title : **Existence and Classification of certain non-integrable distributions**

Abstract : Existence and Classification of geometric structures on manifolds are fundamental interests in differential topology. Sometimes, existence of geometric structures contribute to the global study of the base manifold. In this talk, I would like to introduce some attempts to study geometric structures from the view point of the h-principle. That method relates geometric structures some topological conditions. My struggle will come to $(3,5)$-distribution, Cartan's $(2,3,5)$-distribution, and their generalizations.

15:10–15:30 **Tea time**

(9) 15:30–16:20 **Ohno Soma** (大野走馬), Waseda University

Title : **Rarita-Schwinger fields on nearly parallel G_2-manifolds**

Abstract : Rarita-Schwinger fields play an important role on Physics. On the other hand, in mathematics, Rarita-Schwinger fields can be considered as a "spin-3/2 version" of the harmonic spinor. In this study, we find that the space of Rarita-Schwinger fields coincides with a subspace of an eigenspace of the Laplacian on nearly parallel G_2-manifolds. Applying the same technique to a deformation theory, we also identify the space of infinitesimal deformations of a Killing spinor.

(10) 16:30–17:20 **Homma Yasushi** (本間泰史), Waseda University

Title : **Higgs algebra in harmonic analysis on the Grassmannian of 2-planes**

Abstract : In the classical harmonic analysis on the sphere S^{m-1}, it is known that $X = |x|^2/2$, $Y = -\Delta_x/2$, and $H = E_x + m/2 = \sum x_i\partial_i + m/2$ constitute the Lie algebra $\mathfrak{sl}(2,\mathbb{R})$, which acts on the space of polynomial $\mathcal{P}(\mathbb{R}^m,\mathbb{C})$ on $\mathbb{R}^m$. Then we have a $\mathfrak{sl}(2,\mathbb{R})\times \mathrm{SO}(m)$-decomposition, $\mathcal{P}(\mathbb{R}^m,\mathbb{C}) \cong V^{\infty}_{k+m/2}\otimes \mathcal{H}_k$, where $V^{\infty}_{k+m/2}$ is an (∞-dimensional) irreducible Verma-module of $\mathfrak{sl}(2,\mathbb{R})$ with the lowest weight $k+m/2$ and $\mathcal{H}_k = \mathcal{P}_k \cap \ker \Delta_x$ is an irreducible $\mathrm{SO}(m)$-module of the harmonic polynomial with degree k. Note that the restriction $\mathcal{P}(\mathbb{R}^m,\mathbb{C})$ onto S^{m-1} gives $L^2(S^{m-1},\mathbb{C})$.

We consider a generalization from the sphere to the (oriented) Grassmannian manifold $\mathrm{Gr}_0(m,2)$ of 2-planes in $\mathbb{R}^m$. The harmonic analysis on such manifolds has been investigated by many researchers. In this talk, I will present a new aspect that an algebra called the Higgs algebra appears instead of $\mathfrak{sl}(2,\mathbb{R})$. The algebra H_3 is a polynomial deformation of $\mathfrak{sl}(2,\mathbb{R})$ invented by physicist Higgs (but, no relation to Higgs fileds) and developed in physics. By using H_3, we have a $H_3\times \mathrm{SO}(m)$-decomposition of the polynomial space $\mathcal{P}(\mathbb{R}^{m\times 2},\mathbb{C})^{SL(2,\mathbb{R})}$ whose resection gives $L^2(\mathrm{Gr}_0(m,2),\mathbb{C})$. If I have time, I will also talk about the Casimir of H_3 and the Pizzetti formula on $\mathrm{Gr}_0(m,2)$ as applicaitons. This talk is based on a collaborative research with D. Eelbode in Antwerp university.

Thank you for your cooporation and interest.