Clifford algebras and conformal geometry of surfaces

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Introduction
1. The model case: Two-dimension
2. Projective lightcone model
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