Zeta regularized Determinant of the Laplacian for classes of Spherical Space Forms

joint work with:

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Generalizing the classical Riemann zeta function and in case of a closed compact Riemannian manifold M the spectral zeta function $\zeta_M(\,s\,)$ of the Laplacian Δ_M on M has been introduced by Minakshisundaram and Pleijel. Meromorphic continuations of $\zeta_M(\,s\,)$ to the complex plane have close connections to various aspects of special functions. In particular, the behavior of $\zeta_M(\,s\,)$ in $s = 0$ is of interest in physics, conformal field theory and string theory. Moreover, the zeta regularized determinant of Δ_M can be expressed in terms of the derivative $\zeta'_M(\,0\,)$.

The n-sphere $\mathbb{S}^n$ or the real projective space $\mathbb{R}P^n$ have been frequently studied under such aspects. In this talk we consider special classes of spherical space forms $M := \mathbb{S}^n/G$ where $G \subset SO(n+1)$ is a finite subgroup acting freely on $\mathbb{S}^n$. First, we decompose $\zeta_M(\,s\,)$ in terms of certain Dirichlet series depending on G and $\dim M$. As a result we obtain explicit formulas of the zeta regularized determinant of the Laplacian for classes of lens spaces but our method works more generally. We treat an example where G is not a cyclic group but is induced by the dihedral group. As a crucial ingredient to our analysis and in particular for determining the eigenspace dimensions of Δ_M we use a rational form for the generating function of the multiplicities of eigenvalues of Δ_M due to A. Ikeda. Meromorphic extensions of the Dirichlet series mentioned above are found by applying the so called *Egami interpolation method* to a certain integral expression. We mention that in these calculations *Barne's G-function* appears naturally.

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